Determination of Power System Topological Observability Using Improved Hopfield Neural Network

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ABSTRACT

This paper formulates the power system Topological Observability (TO) problem as an integer programming problem, and develops a new methodology based on Improved Hopfield Neural Network (IHNN) for the determination of TO in power system networks. These complex power systems require accurate and efficient controls that makes the control centers to work efficiently. These control centers are equipped with Supervisory Control and Data Acquisition (SCADA) systems allowing to acquire information about the power system, and its transmission to control centers in real time. The computations in real time environment are reaching a limit, as far as conventional computer based algorithms are concerned. Hence, it is required to find out newer methods for these applications, which can be implemented on hardware to outperform their software counterpart. Therefore, this paper solves the TO problem using the IHNN. This algorithm is based on neural networks and can easily be implemented on dedicated hardware. The proposed approach is implemented on 5 bus, 10 measurement and 9 bus, 14 measurement systems. Some of the results obtained with proposed IHNN are also compared with Genetic Algorithm. The results obtained using the proposed IHNN have greater computational speed with lesser hardware requirement.

Keywords: Topological Observability, Hopfield Neural Networks, Power System Configuration, Power System State Estimation.

1. INTRODUCTION

For the power system operation and control, online data and information are very important. Traditionally, this is accomplished by the state estimators which provide credible data from raw data acquired from the measurement devices. In state estimation applications, system observability is of major concern [1]. Power system Topological Observability (TO) analysis is extremely important to examine whether the relationship between measurement allocation and power system configuration is appropriate. Every time when a system configuration is changed for some reasons, a Topological Observability (TO) test should be executed prior to performing the state estimation, to check one-to-one correspondence between buses and measurements. If this is not satisfied, observability analysis methods can provide the minimum set of additional measurements needed to restore the observability. Therefore, an efficient implementation of TO test is important in achieving the satisfactory performance for the entire state estimation process, and whole real time monitoring and control of power systems.

So far, the observability analysis has been accomplished with the help of numerical/topological approaches. The TO problem can also be formulated as a combinatorial optimization problem. Conventional methods based on graph theory are available, but as the number of buses increases, the computational burden increases drastically [2-4]. In 1975, a method for TO has been developed in [2]. After that, a lot of work has been done in this area. Determination of observability of electrical power system from the point of view of topological observability is developed in Reference [5] using genetic algorithms. The problem can be reduced to the determination of whether a spanning tree that fulfills certain conditions with regards to the use of available measurements exists. A graph theory based method, and the Gauss elimination based approach are developed in References [4, 6]. A technique to solve the TO problem using the maximum flow is developed in [7]. An artificial neural networks (ANNs) based approaches using recurrent networks for solving TO problem are proposed in [8-9]. Observability is the fundamental part of state estimators. Different observability algorithms are used in state estimators like algebraic, numerical, topological and hybrid. A survey of topological observability algorithms have been presented in Reference [1]. An attempt has been made to implement ANN’s for observability determination, State Estimation, Economic Load Dispatch and for Reactive Power Optimization in [10]. The problem of observability of power systems from the point of view of topologi-
cral observability and using genetic algorithms (GA) for its determination is addressed in [11]. A method for determining state values with an Artificial Neural Network (ANN) considering topological observability in power systems is proposed in [12], and it evaluates the pseudo-measurement state values of data which are lost in power systems. References [13-14] present the observability problem from the point of view of topological observability and using genetic algorithms for its determination. Optimal placement of phasor measurement units to make a system completely observable is presented in [15].

In this paper, an Improved Hopfield Neural Networks (IHNNT) are proposed for solving the TO problem. HNN consists of a large number of symmetrically connected neurons, for which an energy function can be defined, and it is specific to a particular application. Therefore, if an optimization problem can be mapped onto the Hopfield type neural network, there is a large possibility that the problem will be solved. In solving the TO problem using the HNNs, the equality constraints can be taken into consideration by adding the corresponding terms to the energy function. However, inequality constraints are indispensable in some cases, and hence they should be handled in the network. In [16], slack variables are introduced to handle the inequality constraints by converting them into equality constraint. However, the slack variables make the problem more complicated by increasing the number of neurons, and thereby increasing the number of local minima. In this paper, a new method based on HNN has been developed for judging the network topological observability (TO), and identifying where the meters should be placed to recover the network TO. The proposed method i.e., Improved HNN (IHNNT) is computationally much simpler compared to the root based technique, which requires complicated programming logic and requires reduced number of neurons in the HNN. This results in greater computational speed and reduction in the hardware circuit requirement. The main advantages of using the IHNNT are (i) the internal parameters of the network are explicitly obtained by the valid-subspace technique of solutions, (ii) lack of need for adjustment of penalty factors for initialization of constraints, (iii) for real time application, the modified Hopfield network offers simplicity of implementation in analog hardware or a neural network processor, and (iv) training and testing of the neural network under human supervision is not required.

The remainder of the paper is organized as follows: The problem formulation is described in Section 2. The description of proposed Improved HNN is presented in Section 3. Section 4 presents the proposed solution methodology for solving the TO problem. Section 5 presents the results and discussion. Finally, Section 6 brings out the conclusions with concluding remarks information about network topology and measurements, were developed in order to avoid the difficulty of numerical computation of the rank of measurement Jacobian matrix. Such algorithms have been used widely in the state estimator observability programs. In the case of networks containing only line flow measurements in which real and reactive measurements occur in pairs, the topological condition for the observability is that there exists at least one bus voltage magnitude measurement, and that a spanning tree of the entire network can be built using only measured lines. Finding such a tree can be done using one of the well known tree search methods such as breadth-first or depth-first search. For a N bus network with only bus injection measurements, the determination of observability is even simpler, there must be at least one bus voltage measurement, and at least \( N - 1 \) bus injection measurements. State vector of an electric network consists of the complex voltages at the buses. Unmeasured tap positions of transformers may also be included into the state vector. A measurement vector consists of power flows, power injections, voltage and current magnitudes and tap positions of transformers. For a N bus system, the state vector \( X = [\delta, V]^T \), of dimension \( n = 2N - 1 \), consists of the \( N - 1 \) bus voltage angles \( \delta_i \) with respect to a reference bus and the \( N \) bus voltage magnitudes \( V_i \) for \( i=1, 2, 3, ..., N \). The measurement model for state estimator is represented by [17-18],

\[
z = h(x) + e\tag{1}
\]

where \( z \) represents all measurements, including power injection, power flow and bus voltage magnitude measurements, \( e \) is measurement noise vector, \( x \) is state vector composed of phase angles and magnitudes of voltages at network buses, \( h(\cdot) \) stands for the non-linear measurement functions relating measurements with state variables. Equation (1) is approximated by a linear plus a constant term model as,

\[
\Delta z = [H] \Delta x + e \tag{2}
\]

where

\[
H = \begin{bmatrix}
H_\theta & 0 \\
0 & H_v
\end{bmatrix}
\]

The \( N \) bus power network is observable with respect to a given measurement set \( M \), if and only if the rank of the gain matrix \( (H) \) is equal to \( 2N - 1 \). By applying the \( P - \theta/Q - V \) decoupling principal, the observability problem can be decoupled into \( P - \theta \) observability and \( Q - V \) observability. A network is said to be \( P - \theta \) observable, if the rank of matrix \( H_\theta \) is equal to \( N - 1 \), and it is said to be \( Q - V \) observable, if rank of \( H_v \) matrix is equal to \( N \).
The TO problem is reformulated as an integer programming problem. The condition that a network is topological observable implies that a bus has at least one measurement and a measurement is assigned to only one bus [9]. Let $W$ be a graph representing relationship between $m$ measurements and $n$ buses. The topological observability (TO) is examined by examining whether the equations (4) and (5) holds good:

$$\sum_{j=1}^{n} w_{ij} = 1 \text{ for } i = 1, 2, ..., m \text{(row constraint)} \quad (4)$$

$$\sum_{j=1}^{n} w_{ij} = 1 \text{ for } j = 1, 2, ..., n \text{(column constraint)} \quad (5)$$

Equation (4) implies that measurement $z_i$ is assigned to only one bus, and the Equation (5) indicates that bus $n_j$ has at least one measurement.

2. IMPROVED HOPFIELD NEURAL NETWORK

Artificial neural networks attempt to achieve good performance via dense interconnection of simple computational elements. Hopfield networks [19] are single-layer networks with feedback connections between the nodes. In the standard case, the nodes are fully connected. The node equation for the continuous-time network with $n$-neurons is represented by [19],

$$u_i(t) = -\eta u_i(t) + \sum_{j=1}^{n} T_{ij} V_j(t) + i^b_i \quad (6)$$

$$v_i(t) = g(u_i(t)) \quad (7)$$

where $u_i(t)$ is the current state of $i^{th}$ neuron, $V_j(t)$ is the output of $j^{th}$ neuron, $i^b$ is the offset bias of $i^{th}$ neuron, $\eta u_i(t)$ is the passive decay term, and $T_{ij}$ is the weight connecting $j^{th}$ neuron to $i^{th}$ neuron.

In Equation (7), $g(u_i(t))$ is a monotonically increasing threshold function that limits the output of each neuron to ensure that network output always lies in or within a hypercube. In Reference [20], it is shown that the equilibrium points of the network correspond to values of $v(t)$ for which the energy function associated with the network is minimized using,

$$E(t) = -\frac{1}{2} v(t)^T T v(t) - v(t)^T i^b \quad (8)$$

Mapping of constrained nonlinear optimization problems using a Hopfield network consists of determining the weight matrix ($T$) and the bias vector ($i^b$) to compute equilibrium points. Some mapping techniques, codes the validity constraints as terms in the energy function which are to be minimized when the constraints ($E^{\text{cons}} i = 0$) are satisfied.

$$E(t) = E^{\text{op}}(t) + b_1 E^{\text{cons}}_1(t) + b_2 E^{\text{cons}}_2(t) + .. \quad (9)$$

where $E^{\text{op}}(t)$ represents the objective function to be optimized and $E^{\text{cons}}$ represents the constraints of the problem. The $b_i$ parameters in Equation (9) are constant weightings given to various energy terms. The multiplicity of terms in the energy function tends to frustrate one another, and success of the network is highly sensitive to the relative values of $b_i$. It has been shown in Reference [20] that the $E^{\text{op}}$ and $E^{\text{cons}}$ terms in Equation (9) can be separated into different subspaces so that they no longer frustrate one another. A modified energy function $E'(t)$ can be defined as,

$$E'(t) = E^{\text{conf}}(t) + E^{\text{op}}(t) \quad (10)$$

where $E^{\text{conf}}(t)$ is a confinement term that groups all the constraints imposed by the problem, and $E^{\text{op}}(t)$ is an optimization term that conducts the network output to the equilibrium points. Therefore, the minimization of $E'(t)$ of the improved Hopfield network is conducted in the following two stages:

a) minimization of the term $E^{\text{conf}}(t)$:

$$E^{\text{conf}}(t) = -\frac{1}{2} v(t)^T T^{\text{conf}} v(t) - v(t)^T i^{\text{conf}} \quad (11)$$

where $v(t)$ is the network output, $T^{\text{conf}}$ is weight matrix and $i^{\text{conf}}$ is the bias vector belonging to $E^{\text{conf}}(t)$.

b) minimization of the term $E^{\text{op}}(t)$:

$$E^{\text{op}}(t) = -\frac{1}{2} v(t)^T T^{\text{op}} v(t) - v(t)^T i^{\text{op}} \quad (12)$$

where $T^{\text{op}}$ is weight matrix and $i^{\text{op}}$ is bias vector belonging to $E^{\text{op}}$. This minimization moves $v(t)$ towards an optimal solution (the equilibrium points). Thus, the operation of the modified Hopfield network can be summarized as combination of three main steps, as shown in Fig. 1.

**Fig. 1: Improved Hopfield Neural Network (IHNN).**
Step 1: Minimization of $E^{conf}$ corresponding to the projection of $v(t)$ in the valid subspace defined by [21-22]:

$$v(t) = T^{conf} v(t) + i^{conf}$$

where $T^{conf}$ is a projection matrix such that $T^{conf} T^{conf} = T^{conf}$ and $i^{conf}$ is defined such that $T^{conf} i^{conf} = 0$. This operation corresponds to an indirect minimization of $E^{conf}(t)$.

Step 2: Application of a nonlinear symmetric ramp activation function constraining $v(t)$ in a hypercube is given by,

$$g_i(v_i) = \begin{cases} v_{i}^{min} & \text{if } v_{i}^{min} > v_i \\ v_i & \text{if } v_{i}^{min} \leq v_i \leq v_{i}^{max} \\ v_{i}^{max} & \text{if } v_i > v_{i}^{max} \end{cases}$$

where $v_i \in [v_{i}^{min}, v_{i}^{max}]$.

Step 3: Minimization of $E^{op}$, which involves updating of $v(t)$ so as to direct it to an optimal solution (defined by Top and $v^{op}$) corresponding to network equilibrium points, which are the solutions for the constrained optimization problems. Using the symmetric ramp activation function and $\eta = 0$, Equation (7) becomes,

$$V(t) = g(u(t)) = u(t)$$

Comparing Equation (6) and Equation (11),

$$\frac{dv}{dt} = -\Delta t \bar{N} E^{op}(v) = \Delta t (T^{op} v + i^{op})$$

$$\Delta v = \Delta t v$$

Therefore, minimization of $E^{op}$ consists of updating $v(t)$ in the opposite direction to the gradient of $E^{op}$. Each iteration has two distinct stages. First, as described in Step 3, $v$ is updated using the gradient of the term $E^{op}$ alone. Second, after each updating, $v$ is directly projected in the valid subspace.

2.1 PROPOSED SOLUTION METHODOLOGY FOR SOLVING THE TOPOLOGICAL OBSERVABILITY PROBLEM

A method using slack variables to handle the inequality constraints is proposed in the Reference [23]. On the contrary, the slack variables result in increasing the number of additional neurons [16], which implies that the original problem becomes more complicated due to increase of local minima points. In this paper, an Improved Hopfield Neural Network (HNN) is used for solving topological observability problem. Variable $W$ can be related to neuron output $V$. Let there be $n$ buses and $m$ measurements in the network, and graph of the network is defined in matrix form, where buses and meters are indexed by $x$ and $i$, respectively.

$$V_{xi} = \begin{cases} 0 & \text{if the bus } x \text{ is not assigned measurement } i \\ 1 & \text{if the bus } x \text{ is assigned measurement } i \end{cases}$$

The objective function is defined as [17-18],

$$E = \frac{A}{2} \sum_{x=1}^{n} \sum_{i=1}^{m} V_{xi} V_{yi} + \frac{B}{2} \left( \sum_{x=1}^{n} \sum_{i=1}^{m} V_{xi} - m \right)^2$$

(18)

The first term in the above equation correspond to the constraint that exact one neuron in each column can output 1. Therefore, we get 0 for this term with a valid solution. The second term in Equation (18) has a minimum value of 0, which is attained, if and only if exactly $m$ of the $m \times n$ output states $(V_{ij})$ have value 1, and the rest 0. $A$ and $B$ are constants reflecting relative importance of terms in the energy function.

2.2 Obtaining weight matrix

The steps for obtaining the weight matrix are presented below:

Step 1: Determine local function of motion $du/dt$ from $E$, such that it always decrease $E$, also by continuous Hopfield method,

$$\frac{du_{xi}}{dt} = \sum_{x_i=1}^{n} \sum_{y_i=1}^{m} T_{xi, yi} V_{yi} + \theta_{xi}$$

(19)

Step 2: Find $W_{xi, yi}, \theta_{xi}$ and $du/dt$.

Step 3: Determine $du_{xi}/dt$ from $E$, such that $dE/dt < 0$.

$$\frac{dE}{dt} = \sum_{x_i} \left( \frac{dE}{dV_{xi}} \right) \left( \frac{dV_{xi}}{du_{xi}} \right) \left( \frac{du_{xi}}{dt} \right)$$

(20)

If $\frac{du_{xi}}{dt} = -\frac{dE}{dt}$, then

$$\frac{dE}{dt} = -\sum \left( \frac{dE}{dV_{xi}} \right)^2 \left( \frac{dV_{xi}}{du_{xi}} \right)^2 < 0$$

(21)

$$\frac{dE}{dV_{xi}} = -A \sum_{y_i=1}^{m} V_{yi} - B \sum_{y_i=1}^{m} V_{yi} - m$$

(22)

Since,

$$\frac{du_{xi}}{dt} = \sum_{x_i=1}^{n} \sum_{y_i=1}^{m} T_{xi, yi} V_{yi} + \theta_{xi}$$

(23)

$$W_{xi, yj} = -A \delta_{ij} (1 - \delta_{xy}) - B$$

(24)

where

$$\delta_{xy} = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$$

(25)

Each node also has a positive bias $\theta_{xi} = C \cdot n$. The above algorithm gives minimum of energy function $E$. 


2.3 Neural Network for handling Inequality Constraints in the Topological Observability (TO) Problem

The standard HNN has a serious shortcoming that it cannot solve a combinatorial optimization problem with inequality constraints. Therefore, the neural network, which has been proposed in [24] to solve a simple linear programming (LP) problem with two variables and four constraints [25], is used with slight modification i.e., the sigmoid characteristic is used instead of the linear relationship in the original neural network for solving the inequality constraints in the TO problem. A linear programming (LP) problem including inequality constraints is defined as follows:

\[
\begin{align*}
\text{Minimize, } P &= AV \\
\text{Subject to, } D_jV &\geq B_j (j = 1 \ldots M) \\
D_j &= [D_{j1}, D_{j2} \ldots D_{jn}]^T
\end{align*}
\]

where the \( D_j \) for each \( j \), contains the \( N \) variable coefficients in a constraint equation, and the \( B_j \) are the bounds. The neural network to solve LP problem [6] is shown in Figure 2. The \( N \) outputs \( (V_j) \) of the left-hand set of amplifiers will represent the values of the variables in the LP problem. The components of \( A \) are proportional to input currents fed into these amplifiers. Outputs \( (\psi_j) \) of the right-hand set of amplifiers represent constraint satisfaction. The output \( (\psi_j) \) of the \( j^{th} \) amplifier on the right-hand side injects current into the input lines of the \( V_i \) variable amplifiers by an amount proportional to \( -D_{ji} \), the negative of the constraint coefficient for the \( i^{th} \) variable in the \( j^{th} \) constraint equation. Each of the constraint \( \psi_j \) amplifiers are fed with a constant current proportional to the \( j^{th} \) bound constant \( (B_j) \), and receives input from the \( j^{th} \) variable amplifier by an amount proportional to \( D_{ji} \). Each of the \( V_i \) amplifiers in the LP network has an input capacitor \( C_{ji} \) and an input resistor \( R_i \) in parallel, which connect the input line to ground. The input-output relations of the \( V_i \) amplifiers are linear, and characterized by a linear function \( g_i \) in the relation \( V_i = g(u_i) \) [25]. The \( \psi_j \) amplifiers have the non-linear input-output relation characterized by \n
\[
\begin{align*}
\psi_j &= f(u_j) \\
u_j &= D_jV - B_j \\
f(u_j) &= 0 \text{ if } u_j \geq 0 \\
f(u_j) &= Ku_j \text{ if } u_j < 0
\end{align*}
\]

Where \( K \) is a positive constant, and function \( f \) give large output to variable amplifier, when the corresponding constraint is not satisfied. The input-output characteristic of variable amplifiers should be changed, so that they take an integer value of 0 or 1.

\[
V_i = g(u_i)
\]

The specific features of this Improved Hopfield Neural Network (IHNN) are: (a) inequality constraints need not be included in its energy function, and (b) it always converges to a solution, which satisfies the inequality constraints.

Connections between the constraint neurons and variable neurons must be determined, so that the output of the former make the outputs of the latter satisfy constraints, if they are violated. In order to realize this, all the inequality constraints are first changed into the form of \( h(V) \geq 0 \), so that they are constrained from their lower limits. Then, the coefficients of the constraints are directly used as connections, and all the constraints are expressed by linear function of \( V \).

2.4 Proposed Algorithm

The algorithm for solving the Topological Observability (TO) of power system is presented below:

**Step 1:** Read the required data, and form \( W \) matrix with the given meter configuration of the system. Assign \( V=W \), where \( V \) denotes the initial state of the network.

**Step 2:** Randomly initialize the activation for the network \((U)\).

**Step 3:** Set the iteration count \( =1\).

**Step 4:** Compute the energy \((E)\) value with the present state of network \(V\).

**Step 5:** Check \( E=0 \). If yes, go to Step 9.

**Step 6:** We denote the activation of neuron in the \( i^{th} \) row and \( j^{th} \) column by \( U_{ij} \), and the output is denoted as \( V_{ij} \). \( \Delta t \) denotes the increment in time, from one cycle to the next, and \( T \) denotes the time constant. Change in \( U_{ij}(n) \), i.e., \( \Delta U_{ij} \), which decreases the energy is given by,

\[
\Delta U_{ij} = \Delta t \left[ \frac{U_{ij}}{T} - A \sum_{k=1}^{N} V_{ik} - B \left( \sum_{p=1}^{N} \sum_{k=p}^{N} V_{pk} - m \right) \right]
\]
After that compute new values of activation using,
\[ U_{ij} (n + 1) = U_{ij} (n) + DU_{ij} (n) \] (36)

Output of neurons with new activation is given by,
\[ V_{ij} (n + 1) = F (U_{ij} (n + 1)) \] (37)

The hard switch function \( F(.) \) is represented by,
\[ F(.) = \begin{cases} 
1 & \text{if } U_{ij} (n + 1) > 0 \\
0 & \text{if } U_{ij} (n + 1) < 0
\end{cases} \] (38)

**Step 7:** Enforce inequality constraint using the procedure given in Section 4.2.
**Step 8:** Increment the iteration count by 1, and check if count is less than maximum number of iterations. If yes, goto Step 4, else problem failed to converge.
**Step 9:** Problem converged, and the output state of neural network denotes the location for optimal meter placement.

### 3. RESULTS AND DISCUSSION

The proposed Improved Hopfield Neural Network (IHNN) for the determination of power system topological observability is tested on 5 bus, 10 measurement and 9 bus, 14 measurement systems. The simulation results and discussion is presented next:

#### 3.1 Results for 5 bus, 10 measurements system

The proposed approach is tested on 5 bus, 10 measurement system as shown in Figure 3. In this figure, \( z_1 \) to \( z_5 \) represents the measurements. Two case studies are performed on this system with the number of measurements (m or z) as 8 and 10. Table 1 describes two case studies depending on the number of measurements. Table 1 also shows the number of iterations for each case.

\[ \text{Fig. 3: Five bus, 10 measurement system.} \]

![Five bus, 10 measurement system](image)

The convergence criterion adopted for this problem is \( E=0 \) or the maximum number of iterations (i.e., 200) are reached. The values of constants considered are \( A=0.5, B=0.04, T=100 \) and \( \Delta t=0.01 \). It is to be noted that the convergence is independent of A, B values, and they are chosen as per our interest [17-18].

#### 3.1.1 Case 1: 5 bus system with 8 measurements

In this case, 8 measurements i.e., \( z_1 \) to \( z_8 \) are considered. The initial states (V) of neural network for 8 measurements for Case 1 are shown in Table 2. The final states/output of neural network for Case 1 is given in Table 3.

#### Table 1: Number of measurements and iterations for two cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Measurements</th>
<th>Number of measurements (m)</th>
<th>Iteration count</th>
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<tbody>
<tr>
<td>1</td>
<td>z1-z8</td>
<td>8</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>z1-z10</td>
<td>10</td>
<td>31</td>
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#### Table 2: Initial states of neural network of size (mXn) for Case 1.

<table>
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<th>m</th>
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<th>2</th>
<th>3</th>
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#### Table 3: Final states of neural network of size (mXn) for Case 1.

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<td>8</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The output of the neurons gives the meter location in case of missing measurements. This in turn reduces the meter requirement for the state estimation process. Fig. 4 shows the bipartite graph in the five bus system.

#### 3.1.2 Case 2: 5 bus system with 10 measurements

The final states/output of neural network for Case 2 is given in Table 4. These states gives the meter locations in case of missing measurements. Convergence of energy function with respect to iterations for Case 1 is shown in Figure 5.

#### Table 4: Final states of neural network of size (mXn) for Case 2.

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The simulation results obtained with the proposed Improved HNN (IHN) algorithm are also compared with the approach using the Genetic Algorithms (GA) [13-14]. The detailed description of evolution and encoding has been presented in [13-14]. Figure 5 also depicts the convergence characteristics (i.e., energy function $V_e$, number of iterations) of GA for Case 2. From this figure 5, it can be observed that the proposed IHNN approach has better convergence characteristics in less number of iterations compared to GA.

### 3.2 Results for 9 bus, 14 measurement system

The proposed IHNN has also been applied on 9 bus, 14 measurement system, and the simulation results are presented next:

The convergence criterion adopted in this proposed method is $E=0$. The values of constants taken are $A=0.5$, $B=-0.04$, $T=100$, $\Delta t=0.01$. In this case, 14 measurements are considered, and the Figure 6 depicts the 9 bus, 14 measurement system. The initial and final states of neural network is given in Tables 5 and 6, respectively. The final states gives the meter locations in case of missing measurements.

### 3.2 Results for 9 bus, 14 measurement system

The problem is converged in 30 iterations, and Figure 7 depicts the convergence of energy function with respect to number of iterations. Execution time re-
Table 6: Final States of Neural Network for Nine bus, 14 measurement system.

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>Initial States of Neural Network of Size (m x n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

![Fig.7: Convergence characteristic for 9 bus, 14 measurement system (Energy function Vs. Number of iterations).](image)

The required for this problem is 0.3209 sec.

4. CONCLUSIONS

In this paper, the Topological Observability (TO) problem has been solved using the Improved Hopfield Neural Networks (IHNN). The main shortcoming of standard HNN is that it cannot solve the combinatorial optimization problem with inequality constraints, and this can easily solved using the proposed IHNN. The proposed method is computationally much simpler compared to rooted based technique, which requires complicated programming logic and requires reduced number of neurons in the HNN. This results in greater computational speed and reduction in the hardware circuit requirement. The proposed approach is tested on 5 bus, 10 measurement and 9 bus, 14 measurement systems, and the results show the efficiency and effectiveness of the proposed approach. Some of the results obtained using the proposed IHNN are also compared with the Genetic Algorithms.

References


