Controlling Chaotic Ferroresonance in Autotransformer connecting Metal Oxide Surge Arrester and Neutral Earth Resistance

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ABSTRACT
This paper investigates the effect of linear core losses, metal oxide surge arrester (MOSA) and neutral earth resistance on the onset of chaotic ferroresonance and controlling these overvoltages in an autotransformer. The transformer is chosen for study has a rating of 50 MVA, 635.1 kV. The magnetization characteristic of the autotransformer is modeled by a nonlinear polynomial. The core loss is modeled by linear core losses and is shown as a linear resistance. Three state of ferroresonance are studied: (i) onset of chaos due to interrupt in switching action including linear core losses of the transformer, (ii) effect of MOSA on the limiting these oscillation, and (iii) completely controlling effect of neutral earth resistance on the ferroresonance overvoltages. Results of simulation have been derived and shown by nonlinear dynamics method such as bifurcation diagrams, phase plane and time domain simulations.

Keywords: Linear Core Losses, Chaos, Bifurcation, Ferroresonance Oscillation, Autotransformers, Metal Oxide Surge Arrester, Neutral Earth Resistance

1. INTRODUCTION
Ferroresonance is initiated by improper switching operation, routine switching, or load shedding involving a high voltage transmission line. It can result in Unpredictable overvoltage and high currents. The prerequisite for ferroresonance is a circuit containing iron core inductance and a capacitance. Kieny first suggested applying chaos to the study of ferroresonance in electric power circuits [1]. He studied the possibility of ferroresonance in power system, particularly in the presence of long capacitive lines as highlighted by occurrences in France in 1982, and produced a bifurcation diagram indicating stable and unstable areas of operation. A special ferroresonance phenomenon on 3-phase66kV VT-generation of 20 Hz zero sequence continuous voltage is given in [2]. Typical cases of ferroresonance are reported in [3], [4], in these papers power transformer and VTs has been investigated due to ferroresonance overvoltage. Digital simulation of transient in power system has been done in [5]. Application of nonlinear dynamics and chaos to ferroresonance in the distribution systems can be found in [6]. The susceptibility of a ferroresonance circuit to a quasi-periodic and frequency locked oscillations has been presented in [7]. In this case, investigation of ferroresonance is done upon the new branch of chaos theory which is quasiperiodic oscillation in the power system and finally ferroresonance appears by this route. Modeling iron core nonlinearities has been illustrated in [8]. Mozaffari is investigated the ferroresonance in power transformer and effect of initial condition on this phenomenon. He analyzed condition of occurring chaos in the transformer and suggested the reduced equivalent circuit for power system including power switch and trans [9],[10]. The mitigating effect of transformer connected in parallel to a MOSA has been illustrated in [11]. Analysis of ferroresonance in voltage transformer has been investigated by Zahawi in [12] and [13]. Analysis of ferroresonance phenomena in the power transformers including neutral resistance effect has been reported in [14]. Ferroresonance conditions associated with a 13 kV voltage regulator during back-feed conditions are given in [15]. Performance of various magnetic core models in comparison with the laboratory test results of a ferroresonance test on a 33 kV voltage transformer investigated in [16]. Mitigating ferroresonance in voltage transformers in ungrounded MV networks has been reported in [17]. An approach for determining the subsystem experiencing and producing a bifurcation in a power system dynamic model has been reported in [18]. Then controlling ferroresonance oscillation has been investigated in [19], [20]. It is shown controlling ferroresonance in voltage transformer including nonlinear core losses by considering circuit breaker shunt resistance effect, and clearly shows the effect of core losses nonlinearity on the system behavior and margin of occurring ferroresonance. In [21], electromagnetic voltage transformer has been studied in the case of nonlinear core losses by applying metal oxide surge arrester in parallel with it and simulations have shown that a change in the value of the equivalent line to ground capacitance, may origi-
nate different types of ferroresonance overvoltage. In all previous studies, the effects of linear core losses, MOSA and neutral earth resistance altogether on occurring and controlling ferroresonance oscillations in the autotransformer are neglected. This paper studies the effect of MOSA and neutral earth resistance on the global behavior of a ferroresonance circuit with linear core losses in an autotransformer.

2. CIRCUIT DESCRIPTIONS AND MODELING

The three-phase diagram for the proposed circuit is shown in Fig. 1.

![Fig.1: System modeling of the power system](image)

The 1100 kV power system line was energized through a bank of three single-phase as reported in autotransformers [22]. Chaotic oscillation occurred in phase A when this phase was switched off on the low-voltage side of the autotransformer; phase C was not yet connected to the transformer at that time [23]. The autotransformer is modeled by a T-equivalent circuit with all impedances referred to the high-voltage side [23]. The magnetization branch is modeled by a nonlinear inductance in parallel with a nonlinear resistance and these represent the nonlinear saturation characteristic ($i_{Lm}$) and nonlinear hysteresis and eddy current characteristics ($v_m - i_{Rm}$), respectively [23]. The iron core saturation characteristic is given by:

$$i_{Lm} = a\lambda + b\lambda^q$$

(1)

The exponent $q$ depends on the degree of saturation. It was show that for proper representation of the saturation characteristics of a power transformer the exponent $q$ may take the values 5, 7, and 11. In this paper, the core loss model is considered as a linear resistance and its value is 750kΩ. Hysteresis curve of this iron core is shown in Fig. 2. The polynomial of order seven and the coefficient $b$ of equation (1) are chosen for the best fit of the saturation region. Fig. 3 shows the comparison between different approximations of saturation region against the true magnetization characteristic that was obtained from field measurement by Dick and Watson [24]. Using a polynomial of order less than eleven to present the magnetization curve might be adequate for small capacity transformers such as voltage transformer, but it does not sharply enough at the knee point to satisfy the magnetization characteristic of modern highly capacity transformers. Because of the nonlinear nature of the transformer magnetizing characteristics, the behavior of the system is extremely sensitive to change in system parameter and initial conditions. A small change in the value of system voltage, capacitance or losses may lead to dramatic change in the behavior of it. A more suitable mathematical language for studying ferroresonance and other nonlinear systems is provided by nonlinear dynamic methods. Mathematical tools that are used in this analysis are phase plan diagram, time domain simulation and bifurcation diagram [19].

The circuit in Fig. 1 can be reduced to a simple form by replacing the dotted part with the thevenin equivalent circuit as shown in Fig. 4. By using the steady-state solution of MATLAB SIMULINK with the data of the 1100 kV transmission line [22], Eth
and $Z_{th}$ were found to be:

$$E_{th} = 130.1\text{kv}; z_{th} = -j1.01243E + 0.5\Omega$$

$$E_{th} = 130.1\text{kv}; z_{th} = j1.01243$$

Fig.4: Thevenin circuit of the power system

Fig.5: Circuit of ferroresonance investigations including MOSA

3. NONLINEAR DYNAMICS AND EQUATION

The resulting circuit to be investigated is shown in Fig. 5 where $Z_{th}$ represents the thevenin impedance and MOSA is added to the initial equivalent circuit. The behavior of this circuit can be described by the following system of nonlinear differential equations. By applying KVL and KCL to the circuit in the case of considering linear core losses effect can be driven:

$$\frac{dv_c}{dt} = \frac{1}{C} \left( a\lambda + b\lambda^3 + \left( \frac{1}{R_m} \right) \frac{d\lambda}{dt} + \left( \frac{1}{k} \right) \alpha \frac{d\lambda}{dt} \right)$$

$$v_{L2} = L_2 \frac{di}{dt} = L_2 \left( d\lambda \frac{dt}{d\lambda} + \frac{1}{R_m} \frac{d^2\lambda}{dt^2} + \left( \frac{1}{k} \right) \alpha \frac{d\lambda}{dt} \right)$$

$$e_{th} = v_c + R_2 \left( \frac{1}{R_m} \frac{d\lambda}{dt} + a\lambda + b\lambda^3 + \left( \frac{1}{k} \right) \alpha \frac{d\lambda}{dt} \right) + \frac{d\lambda}{dt} + L_2 \left( \frac{1}{R_m} \frac{d^2\lambda}{dt^2} + a\lambda + b\lambda^3 + \left( \frac{1}{k} \right) \alpha \frac{d\lambda}{dt} \right)^{-1}$$

Parameters values of the power system are given in table (1). Also, values of nonlinear core index are given in table (2).

Table 1: basic parameters value of the power system

<table>
<thead>
<tr>
<th>$V_{base}$</th>
<th>$I_{base}$</th>
<th>$C_p.u.$</th>
<th>$R_p.u.$</th>
<th>$R_{core}$</th>
<th>$R_{base}$</th>
<th>$L_{sp.u.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>635.1KV</td>
<td>78.723A</td>
<td>0.079</td>
<td>0.0014</td>
<td>556.68</td>
<td>8067Ω</td>
<td>0.0188</td>
</tr>
</tbody>
</table>

Table 2: Different value of $q$ with its coefficient

<table>
<thead>
<tr>
<th>q/coefficient</th>
<th>a</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.0071</td>
<td>0.0034</td>
</tr>
<tr>
<td>7</td>
<td>0.0067</td>
<td>0.001</td>
</tr>
<tr>
<td>11</td>
<td>0.0067</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

4. SYSTEM MODELLING INCLUDING MOSA AND NEUTRAL EARTH RESISTANCE

In this case, the system which was considered for simulation is shown in Fig. 6. The primary purpose of inserting impedance between the star point of a transformer and earth is to limit earth fault current. The value of impedance required is easily calculated to a reasonable approximation by dividing the rated phase voltage by the rated phase current of the transformer. Obviously be extension the value for high impedance earthing is then calculated by dividing the rated phase voltage by the prospective low level of earth fault current required. These resistors will dissipate considerable heat when earth fault current flows and are usually only short term rated (typically 30secs) so as to achieve an economic design. According to earth resistance explanation, in Fig. 6, $R_n$ is the neutral earth resistance.
The differential equation for the circuit in Fig. 6 can be presented as follows:

\[
\epsilon_{th} = v_c + R_2 \left( \frac{1}{R_m} \frac{d\lambda}{dt} + a\lambda + b\lambda^\alpha + \frac{1}{k} \left( \frac{d\lambda}{dt} \right)^\alpha \right) \frac{d\lambda}{dt} + \left( \frac{1}{R_m} \right) + \left( \frac{d\lambda}{dt} \right)^\alpha
\]

\[
+ L_2 \left( \frac{1}{R_m} \frac{d\lambda^2}{dt^2} + a\frac{d\lambda}{dt} + b\lambda^\alpha - 1 \frac{d\lambda}{dt} + \alpha \left( \frac{1}{k} \right) \frac{d\lambda}{dt} \right) + \left( \frac{1}{k} \right) \frac{d\lambda}{dt} + \left( \frac{d\lambda}{dt} \right)^\alpha
\]

\[
+ R_n \left( \frac{1}{R_m} \frac{d\lambda}{dt} + a\lambda + b\lambda^\alpha + \frac{1}{k} \left( \frac{d\lambda}{dt} \right)^\alpha \right)
\]

(9)

where \( \lambda \) is the flux linkage and \( V \) is the voltage of transformer. Typical values for various system parameters has been considered for simulation were kept the same by the case 1, while neutral resistance has been added to the system and its value is given below:

\[ R_{neutral} = 25M\Omega \]

5. SIMULATION RESULTS AND DISCUSSION

Due to the multitude of transformer winding and core configurations, system connections, various sources of capacitance, and the nonlinearities involved, the scenarios under which ferroresonance can occur are seemingly endless [22]. System events that may initiate ferroresonance include single-phase switching or fusing, or loss of system grounding. The ferroresonance circuit in all cases is an applied (or induced) voltage connected to a capacitance in series with a transformer’s magnetizing reactance. In this section, time domain simulations were performed using fourth order Runge-Kutta method and validated against MATLAB SIMULINK. The initial conditions as calculated from steady-state solution of MATLAB are:

\[ \lambda = 0; v_m = 1.67pu; v_c = 1.55pu \]

The circuit in Fig.5 is analyzed by first modeling the core loss as a constant linear resistance.

Simulation has been done in two categories, first: system simulation with linear core losses effect considering MOSA and second: system simulation with linear core losses, MOSA and neutral earth resistance effect. In the first step, nonlinear model of transformer disconnected coil has been considered with \( q = 5 \). Fig. 7 shows the phase plan diagram of power transformer with \( q = 5 \), according to this plot, trajectory of the system has a fundamental resonance behavior and overvoltage voltages of transformer reach up to 2p.u.
Fig. 9 shows the phase plan diagram of overvoltage on transformer with $q = 7$. It is shown when the degree of $q$ increases from 5 to 7, the overvoltage remains in 2p.u, but behavior of the system is changed from fundamental resonances to the subharmonic resonances.

Figs. 11 and 12 show the phase plan diagram and time domain simulating with $q=11$. It is shown that when $q$ has been increased, the nonlinear phenomena in the transformer are begun in the low value of the input voltage. It was show that the chaotic oscillation begins at a value of $(E_p = 2pu)$ for $q = 5$ and $(E_p = 1.5pu)$ for $q = 7, 11$ where represents the amplitude of $e_{in}(t)$. Chaotic overvoltage settling down to the source frequency periodic solution was observed for some values of parameters as shown in Fig.7 for a value of $E_p$ in the chaotic Region. Bifurcation diagrams for the linear model of core loss are presented in Figs. 13, 14 and Fig. 15.

Figure. 10 show the time domain simulation with input voltage of system is 4p.u. This plot shows the subharmonic signal with many subharmonic resonances in it. The circuit in Fig. 5 is analyzed by first modeling the core loss as a constant linear resistance.

Bifurcation diagram is the best tool for studying the nonlinear dynamics systems. By this tool system behavior can be analyzes in the best variation. In Fig. 13 input voltage is increased up to 8p.u and overvoltage on transformer has been analyzed according to the input voltage variation. In the case of bifurcation diagrams, period1 appears in $E = 2p.u$ and by this route bifurcation is take placed. Amplitude of the overvoltage is reaches to 1.8p.u, MOSA successfully decreases the overvoltage to 1.8p.u and chaotic signal is changed to the period1 and period3 oscillation. Fig. 14 shows the system overvoltage with $q = 7$, in this plot, one jump has been occurred in $E = 1.8p.u$ and chaos appear in $E = 4.3p.u$. 

**Fig. 10:** Time domain simulation for $q=7$ considering linear core losses shows subharmonic resonance

**Fig. 11:** phase plane diagram for $q=11$ considering linear core losses shows chaotic resonance

**Fig. 12:** Corresponding time domain signal that includes chaotic motion

**Fig. 13:** Bifurcation diagram for $q=5$ considering linear core losses

**Fig. 14:** Bifurcation diagram for $q=7$ considering linear core loss
6. SYSTEM SIMULATION INCLUDING MOSA AND NEUTRAL EARTH RESISTANCE

Figs. 16 and 17 show the phase plane diagram and time domain simulation in the case of considering including MOSA and neutral earth resistance. By comparing these figures with Figs. 7 and 8, it can be concluded that these resistances can cause ferroresonance drop out.

There are some subharmonic resonance in Fig. 16 and 17, but chaotic behavior is ignored and overvoltage reach to 3p.u. Neutral earth resistance successfully controls the ferroresonance overvoltage and chaotic behavior is changed to the fundamental resonances.

Figs. 18 and 19 show the same plot that has been compared with the corresponding phase plane diagram and time domain simulation has been shown in Figs. 9 and 10.

Another tool that can show manner of the system in vast variation of parameters is bifurcation diagram.

In Fig. 22, voltage of the system has been increased to 8p.u and ferroresonance is controlled by neutral earth resistance. Application of MOSA and neutral earth resistance, this nonlinear oscillation is changed to the period1 and period3 oscillation.

Figs. 22, 23 and 24 show the bifurcation diagram in the case of considering linear core losses of transformer while power system is included the MOSA and neutral earth resistance. When $q = 5$ there is no chaotic oscillation in the system. By considering $q$ up to 7, period5 appears in $Eth=5.5$ p.u as is shown.
controlling Chaotic ferroresonance in Autotransformer connecting Metal Oxide Surge Arrester and Neutral Earth Resistance

7. CONCLUSIONS

The dynamic behavior of a transformer is investigated by nonlinear dynamical methods and chaos. MOSA can decrease the ferroresonance oscillation due to the nonlinearity characteristics of it, but it cannot completely limit this overvoltage. Also nonlinear and chaotic overvoltage can destroy the MOSA if it remains more on the transformer. By connecting neutral earth resistance to the neutral point of the autotransformer, these ferroresonance oscillations are successfully controlled and chaotic behavior is changed to the periodic oscillation. In the case of considering this resistance, system manner shows less sensitivity to the initial conditions.

8. APPENDIX: NOMENCLATURE

- \( a, b, c \): index of phase sequence
- \( h_0, h_1, h_2, h_3, h_4 \): coefficient for core loss nonlinear function
- \( n \): index for the neutral connection
- \( a \): coefficient for linear part of magnetizing curve
- \( b \): coefficient for nonlinear part of magnetizing curve
- \( q \): Index of nonlinearity of the magnetizing curve
- \( Z_{th} \): Thevenin’s equivalent impedance
- \( C \): linear capacitor
- \( R_m \): core loss resistance
References


Hamid Radmanesh was born in 1981. He studied Telecommunication engineering at Malek-Ashtar University of Technology, Tehran, Iran, and received the BSC degree in 2006, also studied electrical engineering at Shahed University Tehran, Iran, and received the MSC degree in 2009. Currently, He is PhD student in Amirkabir University of Technology. His research interests include design and modeling of power electronic converters, drives, transient and chaos in power system apparatus.